LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **NOVEMBER 2024**

PST1MC03 – STATISTICAL MATHEMATICS

Date: 13-11-2024 Dept. No. Max. : 100 Marks

Time: 01:00 pm-04:00 pm

	SECTION A – K1 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
1	**Fill in the blanks**
a)	Let $\{a_n\}$ be a sequence with $a_{n+1} \leq a_n \ \forall\ n \in N$ then $\{a_n\}$ is called a __________________ sequence.
b)	Every monotonically decreasing sequence which is bounded below converges to its _____________.
c)	$\sum \frac{1}{n^p}$ *converges if* _________.
d)	Let A be a 4x4 singular matrix then the rank of the matrix A is ___________.
e)	Let Tr(A) denote trace of the matrix A and then Tr(A) is ________ sum of eigen values.
	SECTION A – K2 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
2	**True or False**
a)	Condition for maximum of a function f(x) at a is that $f'(a) = 0$ *and* $f''(a) > 0$.
b)	A bounded sequence always converges.
c)	Let $a_n = (-1)^n$ then -1 and 1 are the two limit points of the sequence a_n.
d)	The maximum possible value for rank of A where A is a 3x4 matrix is 3.
e)	The number of linearly independent vectors in the matrix $A = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 4 & 1 \\ 0 & 1 & 5 \end{bmatrix}$ is 3.
	SECTION B – K3 (CO2)
	Answer any THREE of the following **(3 x 10 = 30)**
3	(i) Prove that the function $\left(\frac{1}{x}\right)^x, x > 0$ has a maximum at x = 1/e. (6 Marks) (ii) Determine $\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$. (4 Marks)
4	If $f: [a, b] \to R$ *is a bounded function and* $P, P' \in P[a, b]$ *such that* $P \subset P'$ *then prove* $(a)\ L(P, f) \leq L(P', f)$ $(b)\ U(P, f) \geq U(P', f)$ (5+5 Marks)
5	Show that R^n is a vector space over the field F.
6	Discuss various types of quadratic forms and explain the conditions to classify the type of quadratic form based on rank, index, and canonical form.
7	(i) What is an Annihilating polynomial? (2 Marks) (ii) What is meant by Minimal Polynomial of a matrix ? (2 Marks) (iii) Find the minimal polynomial of the following matrix: (6 Marks) $A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}$
	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 12.5 = 25)**
8	(i) Show that the sequence $\{x_n\}$ defined by $x_1=1$, $x_n = \sqrt{2 + x_{n-1}}\ \forall\ n \geq 2$ converges to 2. (5 Marks)

	(ii) Show that $\lim_{n\to\infty} \left[\frac{(n+1)(n+2)\ldots(n+n)}{n^n}\right]^{1/n} = 4/e.$ (5 Marks) (iii) Show that $\lim_{n\to\infty} \frac{(n!)^{1/n}}{n} = \frac{1}{e}$. (2.5 Marks)
9	(i) Discuss the convergence or divergence of the following series: $(a)\ 1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + \cdots$ $(b)\ \sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^n x^n\ , x > 0$ (4+4 Marks) (ii) State Leibnitz's Test on alternating series and establish the convergence, absolute convergence of the following series $1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \cdots$ (p > 0). (4.5 Marks)
10	Reduce the quadratic form $2x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 - 4x_1x_3 - 4x_2x_3$ to the canonical form and hence find the rank, index, signature and the type of the given quadratic form.
11	(i) Discuss Gram-Schmidt orthogonalization process. (4 Marks) (ii) Let B={(4,-1,-2,2) , (8,-1,4,0) , (-1,2,0,-2)} is a linearly independent set in R^4. Let U = Span(B). Determine a orthonormal basis for U using Gram-Schmidt orthogonalization process. (8.5 Marks)
	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 15 = 15)**
12	Explain the following tests for convergence of a series: (i) D'Alembert's Ratio Test (ii) Cauchy's Root Test (iii) Raabe's Test (iv) Logarithmic Test (v) D'Morgan and Bertrand's Test (3+3+3+3+3)
13	(i) Use diagonalization method to determine whether the given matrix A is diagonalizable, if so determine the diagonal matrix D such that D = $P^{-1}AP$ A = $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$. (7 Marks) (ii) Which among the following are linear transformation? Justify your answer. (8 Marks) (a) T(x, y) = x - y (b) T(x, y, z) = (z, x+y) (c) T(x, y) = xy (d) T(x, y, z) = 2x – 3y + 4z
	SECTION E – K6 (CO5)
	Answer any ONE of the following **(1 x 20 = 20)**
14	(i) What are upper and lower Darboux Sums? Explain with an example. (4 Marks) (ii) What is Riemann Integral? Explain upper and lower Riemann Integrals. (4 Marks) (iii) Compare $L(P_1,f)$ with $L(P_2,f)$ and $U(P_1,f)$ with $U(P_2,f)$ for the function f(x) = x on [0,1] where P_1 = {0 , 1/4 , 2/4 , 3/4 , 1} and P_2 = {0 , 1/5, 1/4 , 2/4 , 3/4 , 4/5, 1}. (6 Marks) (iv) $If\ f\ is\ defined\ on\ [0,a] by\ f(x) = x^2\ then\ show\ that$ (6 Marks) $Sup\{L(P,f)\} = Inf\{U(P,f)\} = \frac{a^3}{3}$.
15	(i) Determine the eigen values and eigen vectors of the following matrix: (15 Marks) A = $\begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$ and hence establish spectral decomposition of A. (ii) Show that the vectors (1,-1,0,3) , (-2,1,2,4) , (3,0,0,0) , (4,3,2,-2) are linearly dependent using matrix determinant. (5 Marks)

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